

An Effective Curve Matching Metric for Parameter Identification using Partial Mapping

Katharina Witowski*, Markus Feucht[‡], Nielen Stander[†]

*DYNAmore GmbH, Industriestrasse 2, 70565 Stuttgart, Germany

[‡]Daimler AG, EP/SPB, 71059 Sindelfingen, Germany

[†]Livermore Software Technology Corporation, Livermore CA 94551

Abstract

This paper describes a new method for curve matching essential to the solution of inverse problems represented by system parameter identification. Hysteretic response curves are specifically addressed as a general class. The method is based on Partial Curve Mapping (PCM) of the experiment curve onto the computed curve. This methodology involves a curve matching metric which is computed using the volume between the test curve and the computed curve section. A number of examples are presented to demonstrate the capability. These examples represent hysteretic curves which are impossible to match without mapping.

Introduction

Parameter identification problems are non-linear inverse problems which can be solved using mathematical optimization. System parameter identification is a commonly used feature of LS-OPT[®], especially for the purpose of calibrating material models. The procedure consists of minimizing the mismatch between two curves. These two curves typically consist of a two-dimensional experimental target curve and a computed curve. The computed curve is a variable response, being dependent on the system parameters, e.g. material constants. The two main essential components of an algorithm designed for system identification are (i) the optimization algorithm and (ii) the curve matching metric. The focus of the current paper is on the latter.

The original “MeanSqErr” (MSE) feature in LS-OPT uses the vertical coordinate distance between two specified curves to compute the matching error [1]. In this feature, the mismatch is quantified by the sum of the squares of the distances in the y-coordinate between the target points and the interpolated points on the computed curve. Thus, the mismatch of the abscissa is not explicitly included.

A major difficulty with ordinate-based curve matching is that steep parts of the curve are difficult to incorporate in the matching. Failure material models such as the GISSMO model discussed in Reference [2] have the characteristic of a steep decline of the stress-strain curve towards the end of the curve while steep curves also feature in models in which part of the behavior (the leading part of the curve) is linear.

A related problem with ordinate-based matching is that the ranges of the computed and target curves often do not coincide horizontally so that some of the points are ignored. It may even

happen that at an interim stage of the optimization, the two curves do not share any vertical range overlap (there is not a single vertical line which will cross both the computed and the target curves). This type of problem may cause instability of the computation because it becomes impossible to quantify the error.

A third problem is that hysteretic curves (curves with more than one possible y -value for some of the x -values) cannot be quantified because of the non-uniqueness of the ordinate values of the computed curve with respect to the target curve. I.e. a vertical line may cross the same curve more than once.

These kinds of problems present a strong case for active incorporation of the abscissa into the curve-matching metric and the most reasonable approach (perhaps the only one) seems to be to map one of the curves onto the other, in order to make their comparison unique. Two questions immediately arise, namely (i) how to scale the curves and (ii) how to match two curves of unequal length. Scaling is particularly important since scale changes have an effect on distance metrics which incorporate both coordinates. In many cases (e.g. stress vs. strain) there could be several orders of magnitude difference between the values on the abscissa and those of the ordinate.

The mathematical literature provides some ideas on curve matching approaches. Two commonly used metrics for curve matching are the *Hausdorff* [3] and *Fréchet* [4] distances. The Hausdorff distance measures the mismatch between two point sets so is therefore not suitably general for curve matching as there is no continuous point order. For instance it would not be able to handle a hysteretic curve match. The Fréchet distance is better suited for curve matching because it takes the continuity of the curves into account. The Fréchet distance is formally defined as:

$$Fr(P, Q) = \inf_{\alpha, \beta} \max_{t \in [0,1]} \|P(\alpha(t)) - Q(\beta(t))\|$$

where P and Q are polygonal curves, $t \in [0,1]$ represents a position on each curve. The parameters α and β are used to parameterize the distance whereas we can think of t as “time”. The analogy is that of a dog walking along the one curve and the dog’s owner walking along the other connected by a leash. Both walk continuously and monotonically along the curve from the start point to the end point and can vary their velocities according to α and β . The Fréchet distance is the length of the shortest leash that is sufficient for traversing both curves in this manner.

A further possibility is to compute the volume (area) between the two curves. When both curves are normalized, this would typically yield a small mismatch error ($\epsilon \ll 1$) for two reasonably matching curves.

A problem of curve matching is that the curves are typically of different lengths precluding the mapping of entire curves to one another. A practical reason could be that the test curve, which could be the result of digital output from an experiment, is essentially unedited and therefore contains superfluous points unrelated to the actual behavior of the model. Conversely, it may also be that the test curve represents only part of the response, perhaps because only a partial curve could be obtained from the experiment. In parameter identification this issue becomes particularly critical as curves are typically computed at widely distributed points throughout the

parameter space during the optimization process. This potential disparity of curve length requires partial mapping of the two curves.

The next section outlines the Partial Curve Mapping (PCM) methodology for the purpose of finding the best match between two arbitrary curves. Several examples, some with hysteretic curves, are provided to illustrate the methodology.

Partial Curve Mapping (PCM)

The steps for computing the curve mismatch are described in full detail below. The reader should refer to Figure 1 which shows a test curve (in thick red) mapped on to a computed curve. The prime symbol (') is used to denote the curve on which the test curve is being mapped while the double prime symbol (") is used to denote the finally mapped curve. The test curve is shown inside its smallest bounding box, the boundaries of which are used to normalize the curve. Hence the normalized curve a is in the $[(0,0),(1,1)]$ range.

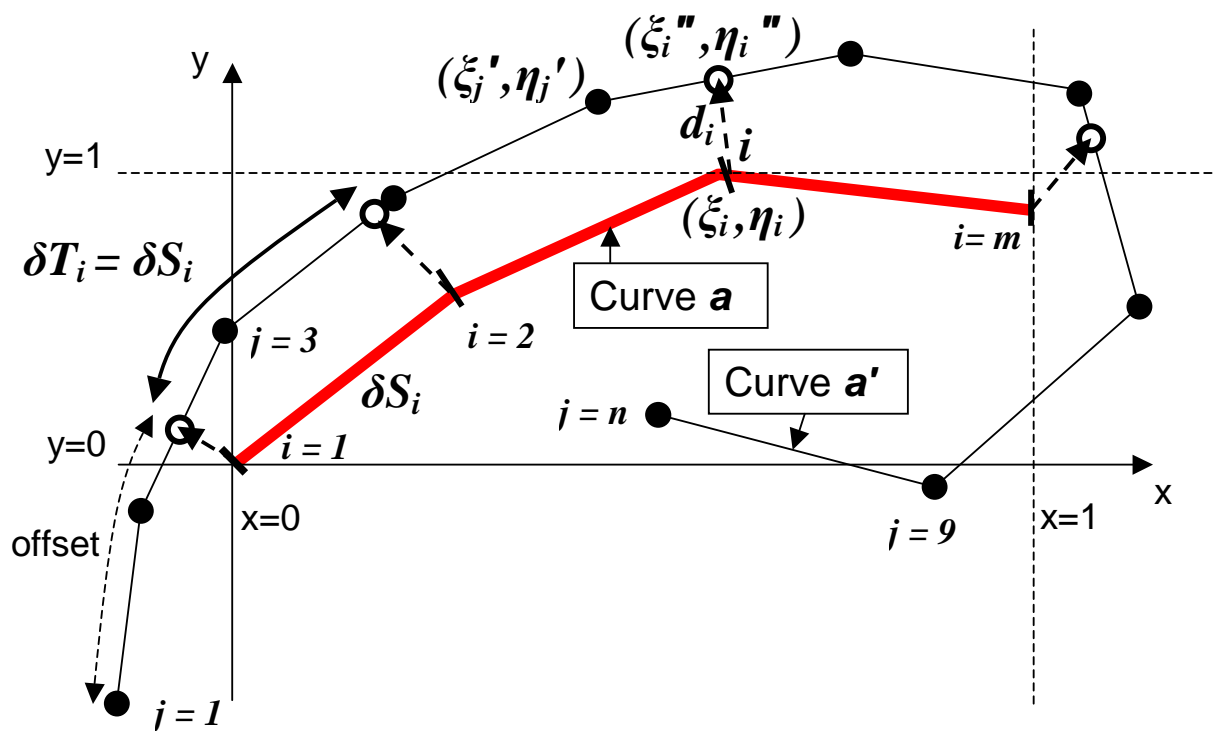


Figure 1: Partial curve mapping of Curve a (in red) to Curve a' with offset. The result is Curve a'' . The solid points represent the original vertices of a' whereas the open circles represent the mapped points representing a'' . Curves a and a' are both normalized to the bounding box of a .

The algorithm for computing the curve mismatch error is as follows:

1. Normalize the m point coordinates i of the *target* curve A to its smallest bounding box to create Curve a . See Figure 1.

$$\xi_i = \frac{X_i - X_{\min}}{X_{\max} - X_{\min}} \quad \eta_i = \frac{Y_i - Y_{\min}}{Y_{\max} - Y_{\min}}$$

$$X_{\min} = \min_k X_k ; \quad X_{\max} = \max_k X_k ; \quad Y_{\min} = \min_k Y_k ; \quad Y_{\max} = \max_k Y_k$$

2. Normalize the n point coordinates j of the *computed* curve A' to the smallest bounding box of A to create curve a' . See Figure 1.

$$\xi_j' = \frac{x_j - X_{\min}}{X_{\max} - X_{\min}} \quad \eta_j' = \frac{y_j - Y_{\min}}{Y_{\max} - Y_{\min}}$$

3. Compute S , the total polygon length of a . Also compute the individual segment lengths δS_i :

$$\delta S_i = \sqrt{(\xi_i - \xi_{i-1})^2 + (\eta_i - \eta_{i-1})^2} ; \quad i = 2, 3, \dots, m$$

Here a segment is defined as a part of the curve between two consecutive points, connected by a straight line.

4. Scale each segment length to the total polygon length S :

$$\tilde{s}_i = \delta S_i / S ; \quad i = 2, 3, \dots, m$$

5. Compute T , the total polygon length of a' .

6. If $S > T$, rename a' to a and a to a' . Hence a will always be shorter than a' .

7. Define an *offset* as a starting point of a curve section of total length S on curve a' . The offset = λ_p will be varied over $p = 1$ to P in order to “slide” Curve a along Curve a' . $\lambda \in [0, T - S]$. Assume P increments in this interval so that each increment has size $\Delta\lambda = \frac{T - S}{P}$.

8. Set $\lambda_p = \lambda_{p-1} + \Delta\lambda$ to create a new section of a' and create point coordinate pairs by mapping each point of curve a to curve a' . A typical curve segment i on a' which corresponds to a segment i on a has length $\delta T_i = \delta S_i$ (see Fig. 1). This creates a new set of point pairs a'' .

9. Compute the discrepancy (mismatch error) between the two curves a and a'' . This is done by summing the *volumes* v_i representing the individual segment errors. First compute the distances between the point pairs:

$$d_i = \sqrt{(\xi_i'' - \xi_i)^2 + (\eta_i'' - \eta_i)^2}$$

Then compute the volume component of each segment. (Note for m points, there are $m-1$ segments.)

$$v_i = \frac{d_i + d_{i-1}}{2} \times \tilde{s}_i; \quad v_1 = 0; \quad i = 2, 3, \dots, m;$$

Then sum the volumes to get the final discrepancy:

$$\varepsilon_p = \sum_{i=1}^m v_i$$

10. Set $p = p+1$ and repeat from point 8.

11. Find the distance $\varepsilon = \min_p \varepsilon_p$. This is the best match between the curves \mathbf{a} and \mathbf{a}' .

Optimization Methodology

In the optimization program LS-OPT[®], the proposed curve matching metric is used to calibrate material or system properties. A finite element model is set up to include a number of parameters which can be changed in order to change the behavior of the model. These parameters are typically material properties. LS-OPT has been programmed to compute the mismatch error as described above and modifies the parameters iteratively in order to minimize the mismatch error value. By minimizing the error value, the model parameters are being calibrated to the experimental results.

The optimization procedure is based on a sequential approximation of the problem leading to a converged optimal result. Firstly an approximation is constructed for each point of the computed curves. For instance, if the test curve is represented by a force vs. a displacement, an approximation is constructed for each point of the force vs. time history as well as for each point of the displacement vs. time history. These approximations are constructed using Response Surface Methodology [5] and generalize the respective histories so that they can be interpolated at any point in the parameter space. Linear approximations are typically used, since they are simple and hence inexpensive to compute. For redundancy and in order to filter any possible noise, the number of simulations per iteration is typically around 1.5 times the number of unknown optimization parameters. Their disadvantage is that, depending on the degree of nonlinearity of the basic responses (e.g. force or displacement), multiple iterations might be necessary. Using the approximations, a cross-plot, in this case a force vs. displacement curve, can thus be constructed for any set of system parameters. The approach is well established and known as the Sequential Response Surface Method (SRS) [1,6].

By exploring the parameter space, the optimizer can find a parameter set in which the curve mismatch can be minimized. The default optimization algorithm used for parameter space exploration in LS-OPT is the Adaptive Simulated Annealing algorithm [7]. Because of the fact

that approximations are used to construct the histories, the solution after the first iteration is also approximate. This can be seen in the optimization histories displayed for the examples below. The points (computed values) (as in Fig. 2(b)) do not always coincide with the line (approximation). Subsequent iterations refine the solution until no further improvement is possible in which case an optimal parameter set has been obtained.

Although the analyst would always desire the mismatch to disappear altogether, this does not always happen. Many factors are involved, such as experimental errors and noise, misrepresentation by the finite element model, discretization errors, incorrectly chosen material model, etc.

Examples

A number of examples were run to test the capability. The nonlinear dynamics finite element program LS-DYNA[8] was used in each case.

Example 1: Hysteretic model (DYNAmore). In the first example five variables were used to calibrate a material model. The results of a force-displacement test were used as a target curve. The material is significantly nonlinear and has a hysteretic behavior in the sense that the loading and unloading curves are distinct (see Figure 2a). The example converged finely in about 3-4 iterations using a sequential linear response surface method. The convergence history is shown in Figure 2b.

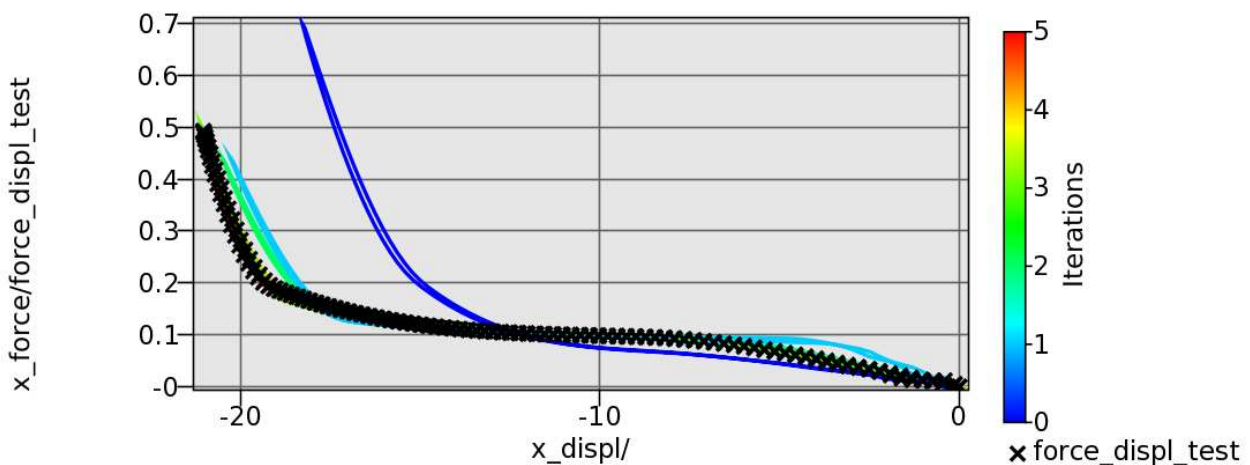


Figure 2 (a) Force-displacement curves at various iterations. The black crosses (+) represent the test curve.

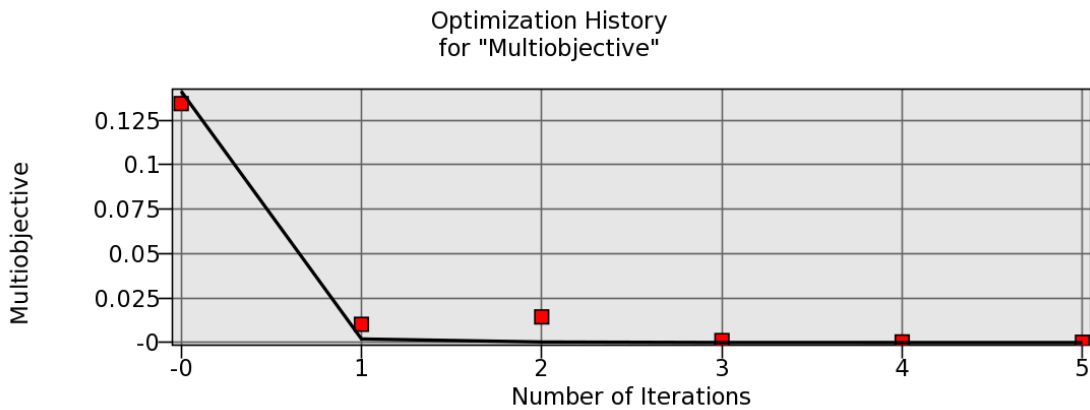


Figure 2 (b) Optimization history of the curve mismatch. The red points represent the actual computed mismatch values while the black line represents the mismatch computed from the approximations.

Example 2: Hysteretic model (TRW). The second example represents a material with a much more prominent hysteresis. Four material parameters are used. The test results were purposefully trimmed to represent only a part of the full hysteresis loop so that the ability of the curve matching algorithm to identify a suitable match using only a part of the data can be tested. As in the first example, a sequential linear approximation was used. The example converged in about 2 iterations, but came very close in a single iteration (light blue curve in Figure 3(a)).

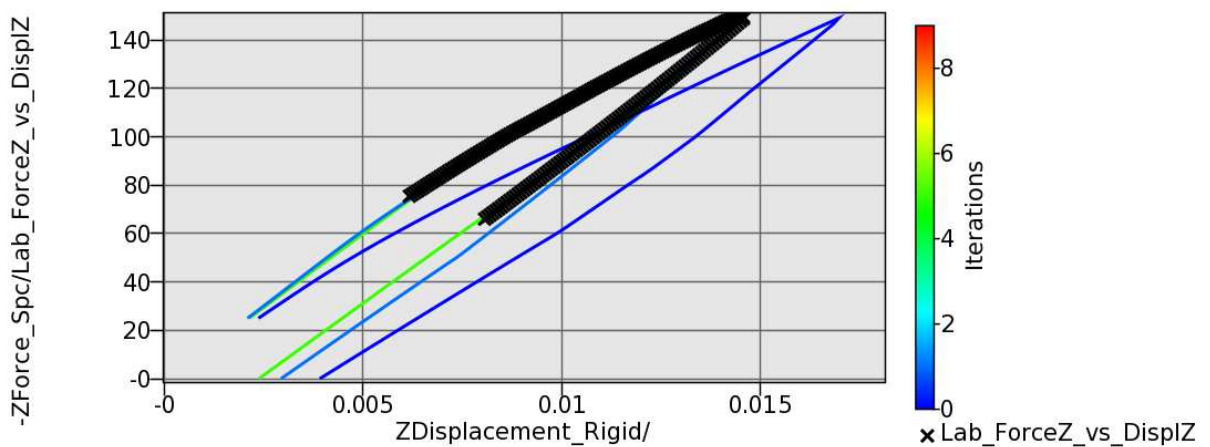


Figure 3 (a): Force-displacement curves at various iterations. The black crosses (+) represent the test curve.

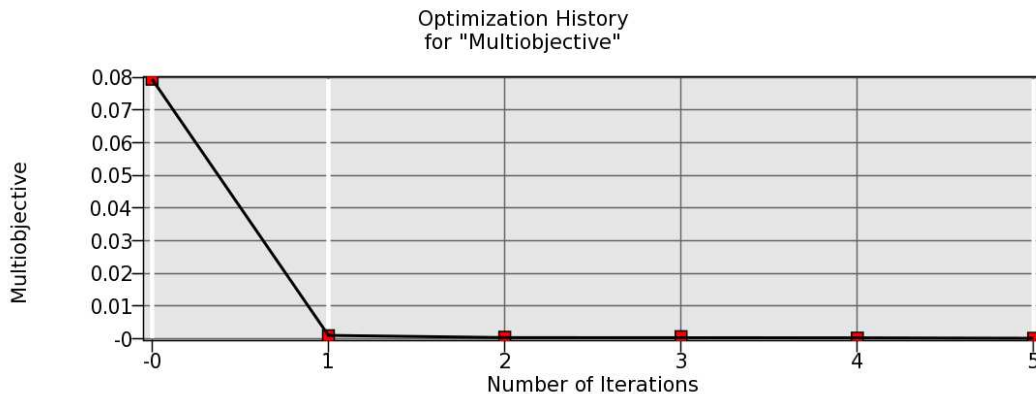


Figure 3 (b): Optimization history of the curve mismatch. The red points represent the actual computed mismatch values while the black line represents the mismatch computed from the approximations (courtesy TRW).

Example 3: GISSMO model (Daimler). With the basic capabilities tested, it was decided to add a practical industrial example. An interesting example, and one of great practical importance to LS-DYNA users, is the calibration of the GISSMO [2] damage model. Because of the failure properties of the model, the GISSMO model is characterized by steep beginning and end sections of the stress-strain curves, so presented a particular challenge.

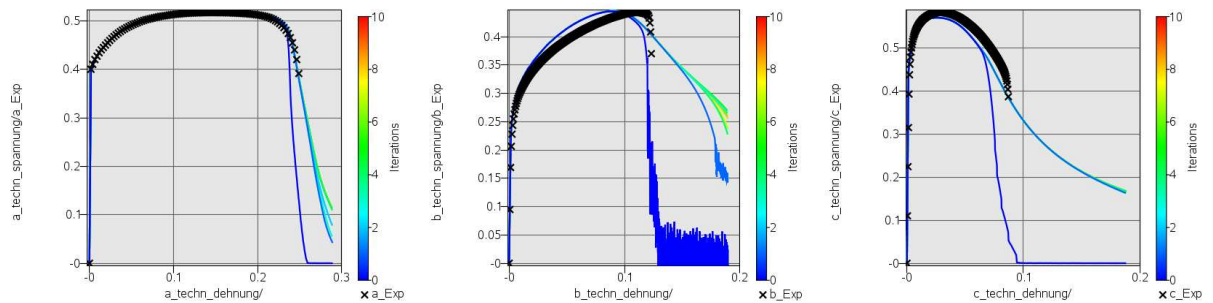
The example consists of 3 test cases and the material has 7 unknown parameters. The objective function was computed by summing the objective functions of the three cases. Two optimization runs were conducted, each using a different starting point. The results are depicted in Figure 4 and Figure 5 respectively.

The results for *Starting Point 1* obtained with the classical MSE method in Figure 4(a) show the well known problem of non-converging behavior due to the very steep slope of the test curve. Note that fewer points are used for the MSE method, since the last few points represent a slight rebounding of the behavior (the abscissa diminishes).

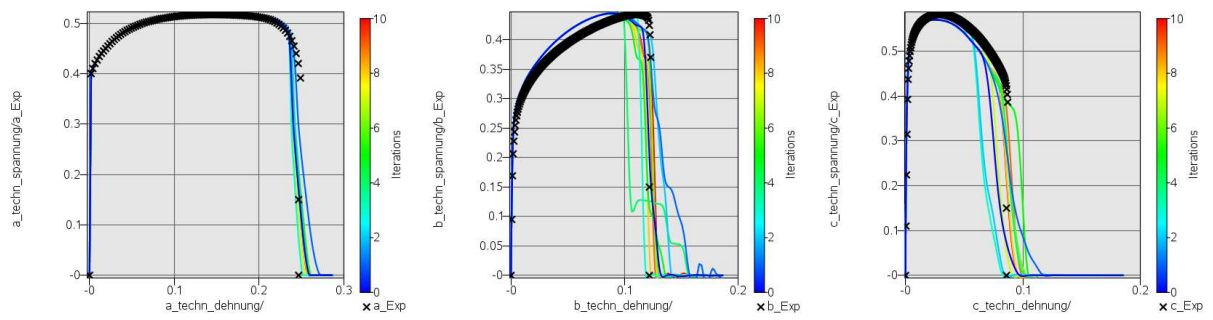
In order to capture correctly the fracture strain of a material, a second optimization was performed using the new PCM method. The result is shown in Figure 4(b), where a significant improvement compared to the MSE result in Figure 4(a) is observed. Since the original damage parameters were close to the optimum, the curve fit was not dramatically improved. However, there was a small improvement after 6 iterations resulting in an almost perfect fit. Figure 4(c) shows the optimization history of the three objectives. It can be seen that the response surfaces (linear in this case) are initially not accurate enough to represent the model, but through iterative refinement, this improves to an almost perfect metamodel fit (Iteration 6).

Starting point 1 (close to optimum)

(a) MeanSqErr (ordinate-based)



(b) Curve Mapping



(c) Curve Mapping (optimization history)

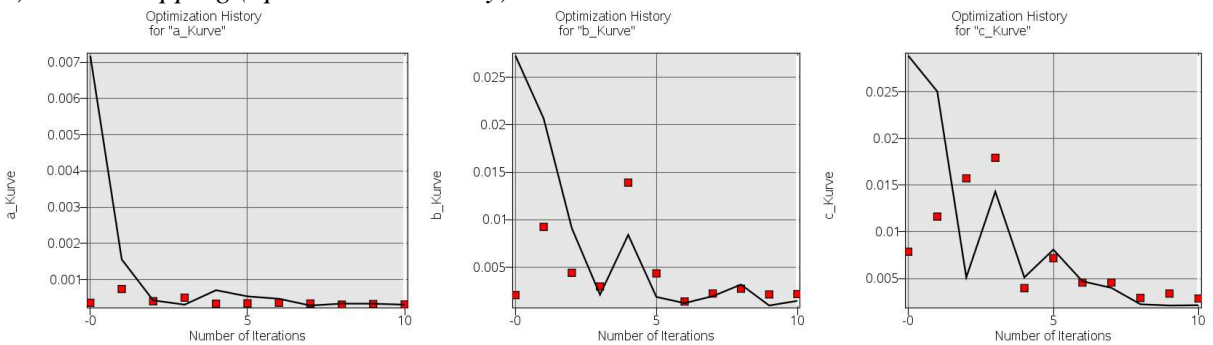
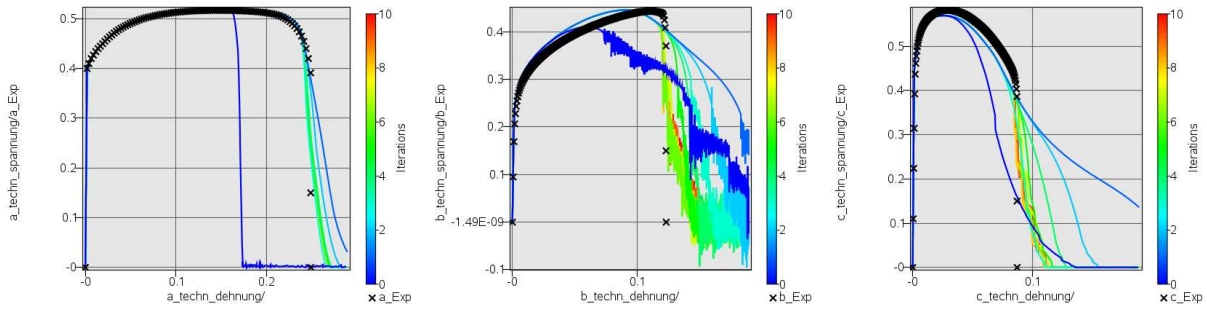


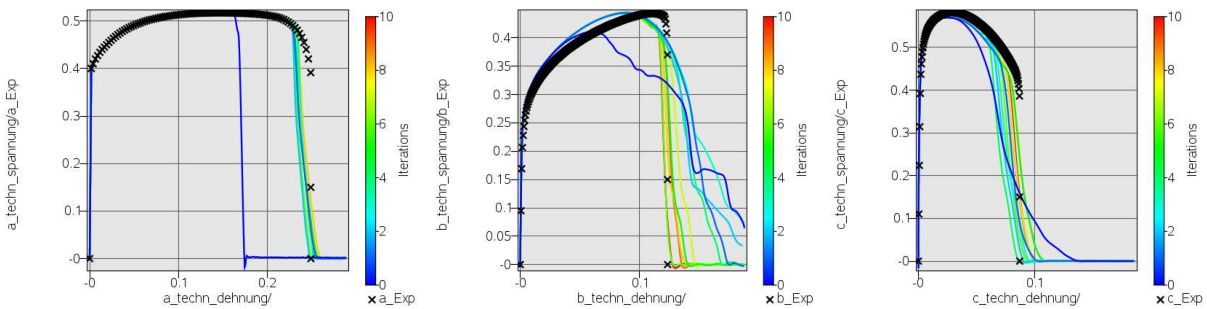
Figure 4: Optimization from a starting point close to the optimum: (a) Optimization of the MSE: Stress-strain curves at various iterations (b) Optimization with PCM: Stress-strain curves at various iterations. The black crosses (+) represent the test curve. (c) PCM Optimization history of the curve mismatch. The red points represent the actual computed mismatch values while the black line represents the mismatch computed from the approximations (courtesy Daimler AG).

Starting point 2 (remote)

(a) MeanSqErr (ordinate-based)



(b) Curve Mapping



(c) Curve Mapping (optimization history)

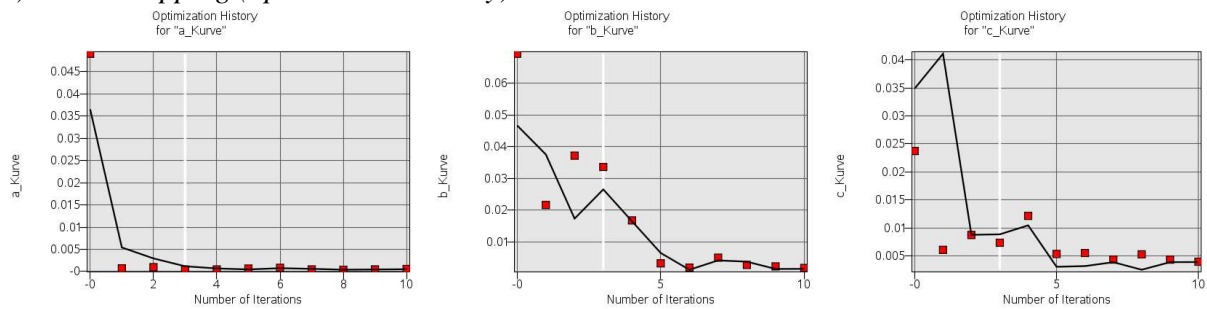


Figure 5: Optimization from a starting point far away from the optimum: (a) Optimization of the MSE: Stress-strain curves at various iterations (b) Optimization with PCM: Stress-strain curves at various iterations. The black crosses (+) represent the test curve. (c) PCM Optimization history of the curve mismatch. The red points represent the actual computed mismatch values while the black line represents the mismatch computed from the approximations (courtesy Daimler AG).

The same example was attempted with a different starting point, this time far away from the optimum. Another difference in this case is that the last few experimental points (represented by the black crosses) were moved slightly to the right to avoid the rebound behavior that could

present problems for the ordinate-based method. This seems to have helped with the calibration, but, for obvious reasons, cannot be used as a general remedy. Again, the PCM method is an improvement, especially with regard to Case b (see Figures 5 (a) and (b), center plot).

Closure

A new Partial Curve Mapping (PCM) method for matching response curves to experimental curves is proposed and has been implemented in the LS-OPT optimization program. The purpose of the method is the solution of inverse problems such as material identification problems in which an experimental curve is provided to which a computed curve needs to be matched. There are multiple, interrelated advantages of the new metric:

1. It is able to identify hysteretic curves through a continuous mapping procedure.
2. Through partial mapping, an optimal section of the computed/test curve corresponding to a short test/computed curve can be identified.
3. By the nature of the mapping method, the metric incorporates both the ordinate and the abscissa into the mismatch computation.
4. By tracking the abscissa, steep, almost vertical sections of stress-strain curves are automatically incorporated by the mapping procedure.
5. Curve normalization assures that the method is independent of the chosen measurement units.
6. The input is very simple, requiring only the names of the two histories compared.

The method appears to be effective as demonstrated by analyzing two examples with strongly hysteretic behavior. Two of the examples, which includes an industrial example, involve a partial test point set covering only a section of the computed output curve.

The PCM method represents a significant improvement of the parameter identification tools available in LS-OPT and is a major step forward in the identification of failure behavior in material models in general.

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